Position & Source: Position vector $\vec{r}$, source vector $\vec{r}'$, separation vector $\overrightarrow{\Delta r} = \vec{r} - \vec{r}'$

Fundamental Theorems of Vector Calculus:

$$\int_{\vec{a}}^{\vec{b}} \nabla f \cdot \overrightarrow{dl} = f(\vec{b}) - f(\vec{a}) \qquad \int \nabla \cdot \vec{A}\, d\tau = \oint \vec{A} \cdot \overrightarrow{da} \qquad \int (\nabla\times\vec{A}) \cdot \overrightarrow{da} = \oint \vec{A} \cdot \overrightarrow{dl}$$

Cartesian Coordinates: $\overrightarrow{dl} = dx\hat{x} + dy\hat{y} + dz\hat{z} \qquad d\tau = dx\, dy\, dz$

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} \qquad \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{z} \qquad \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical Coordinates: $x = r\, sin\theta\, cos\phi,\ y = r\, sin\theta\, sin\phi,\ z = r\, cos\theta$

$$\overrightarrow{dl} = dr\hat{r} + r\, d\theta\hat{\theta} + r\, sin\theta\, d\phi\hat{\phi} \qquad d\tau = r^2 sin\theta\, dr\, d\theta\, d\phi$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\, sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi} \qquad \nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\, sin\theta}\frac{\partial(sin\theta\, A_\theta)}{\partial \theta} + \frac{1}{r\, sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla\times\vec{A} = \frac{1}{r\, sin\theta}\left(\frac{\partial(sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right)\hat{r} + \frac{1}{r}\left(\frac{1}{sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial(r\, A_\phi)}{\partial r}\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial(r\, A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\hat{\phi}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

Cylindrical Coordinates: $x = s\, cos\phi,\ y = s\, sin\phi,\ z = z$

$$\overrightarrow{dl} = ds\hat{s} + s\, d\phi\hat{\phi} + dz\hat{z} \qquad d\tau = s\, ds\, d\phi\, dz$$

$$\nabla f = \frac{\partial f}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z} \qquad \nabla \cdot \vec{A} = \frac{1}{s}\frac{\partial(s\, A_s)}{\partial s} + \frac{1}{s}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\vec{A} = \left(\frac{1}{s}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial(s\, A_\phi)}{\partial s} - \frac{\partial A_s}{\partial \phi}\right)\hat{z} \qquad \nabla^2 f = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial f}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Tensor Math: $(\vec{a} \cdot \overleftrightarrow{T})_j = \sum_i a_i T_{ij} \qquad (\overleftrightarrow{T} \cdot \vec{a})_j = \sum_i T_{ji} a_i$

Lorentz Force: $\vec{F} = Q(\vec{E} + \vec{v}\times\vec{B})$, On Wire: $\vec{F}_{mag} = \int I(\overrightarrow{dl}\times\vec{B})$

Maxwell's Equations:

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \oint \vec{E} \cdot \overrightarrow{dl} = -\frac{d}{dt}\int \vec{B} \cdot \overrightarrow{da}$$

$$\nabla \cdot \vec{E} = \rho/\varepsilon_0 \qquad \oint \vec{E} \cdot \overrightarrow{da} = Q_{enc}/\varepsilon_0$$

$$\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \qquad \oint \vec{B} \cdot \overrightarrow{dl} = \mu_0 I_{enc} + \mu_0\varepsilon_0\frac{d}{dt}\int \vec{E} \cdot \overrightarrow{da}$$

$$\nabla \cdot \vec{B} = 0 \qquad \oint \vec{B} \cdot \overrightarrow{da} = 0$$

Fields in Matter: $\vec{P} = \vec{p}/volume$ $\quad \vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ $\quad \sigma_b = \vec{P} \cdot \hat{n}$ $\quad \rho_b = -\nabla \cdot \vec{P}$ $\quad \vec{J_p} = \frac{\partial \vec{P}}{\partial t}$

$\vec{M} = \vec{m}/volume$ $\quad \vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$ $\quad \vec{K_b} = \vec{M} \times \hat{n}$ $\quad \vec{J_b} = \nabla \times \vec{M}$

$\nabla \cdot \vec{D} = \rho_f$ $\qquad \oint \vec{D} \cdot \vec{da} = Q_{f\_enc}$

$\nabla \times \vec{H} = \vec{J_f} + \frac{\partial \vec{D}}{\partial t}$ $\qquad \oint \vec{H} \cdot \vec{dl} = I_{f_{enc}} + \frac{d}{dt}\int \vec{D} \cdot \vec{da}$

Linear Materials: $\vec{P} = \varepsilon_0 \chi_e \vec{E}$ $\qquad \vec{D} = \varepsilon \vec{E} = (1 + \chi_e)\varepsilon_0 \vec{E} = \varepsilon_r \varepsilon_0 \vec{E}$

$\vec{M} = \chi_m \vec{H}$ $\qquad \vec{B} = \mu \vec{H} = (1 + \chi_m)\mu_0 \vec{H}$

Boundary Conditions: $\Delta D_\perp = \sigma_f$ $\qquad \Delta \vec{E}_{||} = 0$ $\qquad \Delta \vec{D}_{||} = \Delta \vec{P}_{||}$

$\Delta \vec{H}_{||} = \vec{K}_f \times \hat{n}$ $\qquad \Delta B_\perp = 0$ $\qquad \Delta H_\perp = -\Delta M_\perp$

Ohm's Law and EMF: $\vec{J} = \sigma \vec{E}$ $\qquad \mathcal{E} = \oint (\vec{F}/q) \cdot \vec{dl}$ $\qquad \mathcal{E}_{motional} = -\frac{d\Phi_B}{dt}$

Inductance: $M = \frac{\Phi_{B2}}{I_1} = \frac{\Phi_{B1}}{I_2}$ $\qquad L = \frac{\Phi_{B1}}{I_1}$ $\qquad \mathcal{E}_{induced} = -L\frac{dI}{dt}$ $\qquad \frac{dW}{dt} = \mathcal{E} I$

Continuity of Charge/Current: $\frac{\partial \rho}{\partial t} = -\nabla \cdot \vec{J}$ $\qquad \frac{\partial \rho_b}{\partial t} = -\nabla \cdot \vec{J_p}$

Energy & Momentum: $\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)$ $\qquad u_{EM} = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right)$ $\qquad U_{EM} = \int u_{EM}\, d\tau$

$T_{ij} = \varepsilon_0\left(E_i E_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{\mu_0}(B_i B_j - \frac{1}{2}\delta_{ij}B^2)$ $\qquad \vec{g} = \varepsilon_0\left(\vec{E} \times \vec{B}\right) = \mu_0 \varepsilon_0 \vec{S}$ $\qquad \vec{p}_{EM} = \int \vec{g}\, d\tau$

$\frac{dW}{dt} = -\oint \vec{S} \cdot \vec{da} - \frac{dU_{EM}}{dt}$ $\qquad \frac{\partial u_{EM}}{\partial t} = -\nabla \cdot \vec{S} \quad if \; \frac{dW}{dt} = 0$

$\vec{F} = \frac{d\vec{p}_{mech}}{dt} = \oint \overleftrightarrow{T} \cdot \vec{da} - \frac{d\vec{p}_{EM}}{dt}$ $\qquad \vec{f} = \nabla \cdot \overleftrightarrow{T} - \frac{\partial \vec{g}}{\partial t}$ $\qquad \frac{\partial \vec{g}}{\partial t} = \nabla \cdot \overleftrightarrow{T} \quad if \; \vec{f} = 0$

EM Waves:

Complex : $\vec{E}(\vec{r}, t) = \widetilde{E_0} \exp\left(i(\vec{k} \cdot \vec{r} - \omega t)\right) \hat{n}$ $\qquad \vec{B}(\vec{r}, t) = \frac{k}{\omega}\widetilde{E_0} \exp\left(i(\vec{k} \cdot \vec{r} - \omega t)\right) \left(\hat{k} \times \hat{n}\right)$

Real: $\vec{E}(\vec{r}, t) = E_0 \cos\left(\vec{k} \cdot \vec{r} - \omega t + \delta\right) \hat{n}$ $\qquad \vec{B}(\vec{r}, t) = \frac{k}{\omega} E_0 \cos\left(\vec{k} \cdot \vec{r} - \omega t + \delta\right) \left(\hat{k} \times \hat{n}\right)$

$\frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ $\qquad \langle u \rangle = \frac{1}{2}\varepsilon_0 {E_0}^2$ $\qquad \langle \vec{S} \rangle = c\langle u \rangle \hat{k} = I\hat{k} = \frac{1}{2}c\varepsilon_0 {E_0}^2 \hat{k}$ $\qquad \langle \vec{g} \rangle = \frac{\langle \vec{S} \rangle}{c^2} = \frac{1}{2c}\varepsilon_0 {E_0}^2 \hat{k}$

$P = \frac{I}{c} = \frac{\langle S \rangle}{c}$